MATHEMATICS - Code No. 041

Class-XII-(2025-26)

SET: 1

Time allowed: 3 Hrs. **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided in to 5 sections - A, B, C, D and E
3. Section A comprises of 20 MCQ type questions of 1 mark each.
4. Section B comprises of 5 Very Short Answer Type Questions of 2 marks each
5. Section C comprises of 6 Short Answer Type Questions of 3 marks each.
6. Section D comprises of 4 Long Answer Type Questions of 5 marks each.
7. Section E comprises of 3 source based / case based / passage-based questions (4 marks each) with sub parts.
8. Internal choice has been provided for certain questions
9. This question paper contains 6 pages

1	d) 17
2	d) any point on the line segment joining the points (0, 2) and (3, 0).
3	a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
4	d) 5
5	d) 3
6	c) 16
7	b) $\frac{2}{9}$
8	c) $\sin^2 x$
9	b) -25
10	b) $\vec{r} = (2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}) + \lambda\,(3\hat{\imath} - 2\hat{\jmath} + 6\hat{k})$
11	c) π
12	b) $\frac{1}{2}$
13	b) $\frac{\pi}{3}$
14	d) 46
15	d) $\frac{10}{\sqrt{6}}$
16	c) 6
17	a) 80π
18	a) linear
19	(c) A is true but R is false.
20	(d) A is false but R is true.

21	Find $\lvert\vec{x}\rvert$ if $(\vec{x}-\vec{a})(\vec{x}+\vec{a})=12$, where $\vec{a}$ is a unit vector. Ans: $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=12$ $\Rightarrow \vec{x}\cdot\vec{x}+\vec{x}\cdot\vec{a}-\vec{a}\cdot\vec{x}-\vec{a}\cdot\vec{a}=12$ $\Rightarrow \lvert\vec{x}\rvert^2-\lvert\vec{a}\rvert^2=12$ $\Rightarrow \lvert\vec{x}\rvert^2-1=12\left[\because\ \lvert\vec{a}\rvert=1\ \text{as}\ \vec{a}\ \text{is a unit vector}\right]$ $\Rightarrow \lvert\vec{x}\rvert^2=13$ $\therefore\ \lvert\vec{x}\rvert=\sqrt{13}$ (1+1)	- OR - $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=0 \Rightarrow\ \lvert\vec{a}\rvert^2-\lvert\vec{b}\rvert^2=0$ $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert \Rightarrow\ \sqrt{25+1+49}=\sqrt{1+1+\lambda^2}$ $\sqrt{75}=\sqrt{\lambda^2+2} \Rightarrow \lambda^2+2=75 \Rightarrow \lambda^2=75-2=73$ $\lambda=\pm\sqrt{73}$ (1+1)
22	$\lim_{x\to0} f(x)=\lim_{x\to0}\frac{1-\cos kx}{x\sin x} \Rightarrow \lim_{x\to0} f(x)=\lim_{x\to0}\frac{2\sin^2\left(\frac{kx}{2}\right)}{x\sin x}$ $\lim_{x\to0} f(x)=\lim_{x\to0}\frac{2\sin^2\frac{\frac{kx}{2}}{\left(\frac{kx}{2}\right)^2}\times\left(\frac{kx}{2}\right)^2}{x\frac{\sin x}{x}\times x}=\frac{\frac{2k^2}{4}.\lim_{\frac{kx}{2}\to0}\left(\frac{\sin\frac{kx}{2}}{\frac{kx}{2}}\right)^2}{\lim_{x\to0}\frac{\sin x}{x}}=\frac{k^2}{2}\times\frac{1}{1}=\frac{k^2}{2}$ (1+1) $f(0)=\frac{1}{2}$ Since $f(x)$ is continuous at $x=0$ $\lim_{x\to0} f(x)=f(0) \Rightarrow \frac{k^2}{2}=\frac{1}{2} \Rightarrow k^2=1 \quad \therefore \quad \boldsymbol{k=\pm1}$	
23	$\frac{dy}{dx}=x^{sinx}\left(\frac{sinx}{x}+cosx.logx\right)+2^{sinx}log2.cosx$ (1+1)	
24	Domain: $[-2,-\sqrt{2}]\cup[\sqrt{2},2]$ (1+1)	
25	$\int\frac{1}{9x^2+6x+5}dx=\frac{1}{9}\int\frac{1}{x^2+\frac{6}{9}x+\frac{5}{9}}dx$ $=\frac{1}{9}\int\frac{1}{x^2+\frac{2}{3}x+\frac{5}{9}+\left(\frac{1}{3}\right)^2-\left(\frac{1}{3}\right)^2}dx=\frac{1}{9}\int\frac{1}{\left(x+\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2}dx$, putting $x+\frac{1}{3}=t \Rightarrow dx=dt$ $=\frac{1}{9}\int\frac{1}{t^2+\left(\frac{2}{3}\right)^2}dt=\frac{1}{9}\cdot\frac{1}{\frac{2}{3}}\tan^{-1}\left(\frac{t}{2/3}\right)+C=\frac{1}{6}\tan^{-1}\left[\frac{3\left(x+\frac{1}{3}\right)}{2}\right]+C=\frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right)+C$ (1+1)	
	- OR - $I_3=\int_1^4\lvert x-3\rvert dx$ $\quad x-3\geq0$ for $3\leq x\leq4$ $\quad x-3\leq0$ for $1\leq x\leq3$ $\therefore I_3=\int_1^3(3-x)dx+\int_3^4(x-3)dx$ $=\left[3x-\frac{x^2}{2}\right]_1^3+\left[\frac{x^2}{2}-3x\right]_3^4=\left[9-\frac{9}{2}-3+\frac{1}{2}\right]+\left[8-12-\frac{9}{2}+9\right]=\frac{5}{2}$ (1+1)	
26	Ans: Given that $x = a\sin2t\,(1+\cos2t)$ and $y = b\cos2t(1-\cos2t)$ $\frac{dx}{dt}=a[\cos2t\times2(1+\cos2t)+\sin2t(-\sin2t)\times2]$ $=2a[\cos2t\,(1+\cos2t)-\sin2t]=2a[\cos2t+\cos^2 2t-\sin^2 2t)=2a(\cos2t+\cos4t)$ $\frac{dy}{dt}=b[-\sin2t\times2(1-\cos2t)+\cos2t(2\times\sin2t)$ $=2b[-\sin2t+2\sin2t\cos2t)=2b(\sin4t-\sin2t)$ (1+1+1)	

$$\therefore \left(\frac{dy}{dx}\right) = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2b(\sin 4t - \sin 2t)}{2a(\cos 2t + \cos 4t)} \quad \left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{b}{a}\,\frac{\sin\pi - \sin\frac{\pi}{2}}{\cos\frac{\pi}{2} + \cos\pi} = \frac{b}{a}\times\frac{(-1)}{-1} = \frac{b}{a}$$

\- OR -

Here, $f(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$

$\Rightarrow \log f(x) = \log(1 + x) + \log(1 + x^2) + \log(1 + x^4) + \log(1 + x^8)$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}$$

$$\Rightarrow f'(x) = f(x)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

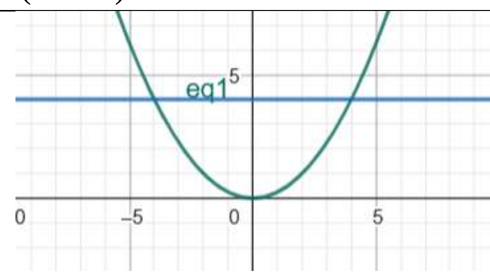
$$\Rightarrow f'(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

Now, $f'(1) = (1 + 1)(1 + 1)(1 + 1)(1 + 1)\left[\frac{1}{1+1} + \frac{2\times 1}{1+1} + \frac{4\times 1}{1+1} + \frac{8\times 1}{1+1}\right]$

$$= [2 \times 2 \times 2 \times 2]\left[\frac{1}{2} + 1 + 2 + 4\right] = 120$$

(1+1+1)

27



Required area = $2\int_0^3 2\sqrt{y}\,dy = 4\left[\frac{2}{3}(3\sqrt{3}) - 0\right] = 8\sqrt{3})sq\ units$

(1+1+1)

28

Given, $y = \sqrt{4 - x^2}$ $\Rightarrow x^2 + y^2 = 4$

$\sqrt{3}x = \sqrt{4 - x^2}$

$\Rightarrow 4x^2 = 4$ $\Rightarrow x = \pm 1$ $\therefore y = \sqrt{3}$

$\therefore$ Coordinates of A is $\left(1, \sqrt{3}\right)$

(1+1+1)

$$\therefore \text{Required Area} = \int_0^{\sqrt{3}} \frac{y}{\sqrt{3}}dy + \int_{\sqrt{3}}^{2}\sqrt{4 - y^2}\,dy$$

$$= \frac{1}{\sqrt{3}}\left[\frac{y^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{y}{2}\sqrt{4 - y^2} + \frac{4}{2}\sin^{-1}\left(\frac{y}{2}\right)\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{\sqrt{3}}\left[\frac{3}{2} - 0\right] + \left[2\sin^{-1}(1) - \frac{\sqrt{3}}{2} - 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{\sqrt{3}}{2} + 2\times\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2\times\frac{\pi}{3} = \pi - \frac{2\pi}{3} = \frac{\pi}{3}\text{ sq. units.}$$

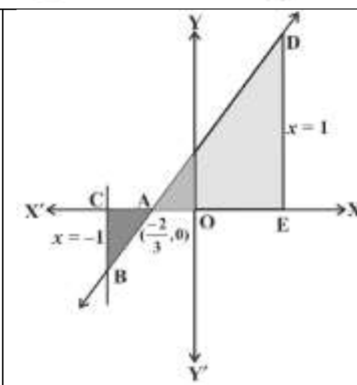
(OR)

$y = 3x + 2$ meets x-axis at $x = \frac{-2}{3}$ and its graph

lies below x-axis for $x \in \left(-1, \frac{-2}{3}\right)$ and above x-axis for $x \in \left(\frac{-2}{3}, 1\right)$

The required area = Area of the region ACBA + Area of the region ADEA

$$= \left|\int_{-1}^{\frac{-2}{3}}(3x+2)dx\right| + \int_{\frac{-2}{3}}^{1}(3x+2)dx = \left|\left[\frac{3x^2}{2} + 2x\right]_{-1}^{\frac{-2}{3}}\right| + \left[\frac{3x^2}{2} + 2x\right]_{\frac{-2}{3}}^{1} = \frac{1}{6} + \frac{25}{6} = \frac{13}{3}$$



(1+1+1)

29

Ans: Given, the equation of a line is: $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda$ (say) $\Rightarrow x = 3\lambda - 2,\ y = 2\lambda - 1,\ z = 2\lambda + 3$

So, we have a point on the line is: $Q(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$ …….(i)

Now, given that distance between two points P(1, 3, 3) and Q(3λ – 2, 2λ – 1, 2λ + 3) is 5 unit i.e. PQ = 5

$\Rightarrow \sqrt{[(3\lambda-2-1)^2+(2\lambda-1-3)^2+(2\lambda+3-3)^2]} = 5$

On Squaring both sides, we get $(3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2 = 25$

$\Rightarrow 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2 = 25$ $\therefore \lambda = 0$ or 2

On putting λ = 0 and λ = 2 in equation (i), we get the required point as (-2, -1, 3) or (4, 3, 7) (1+1+1)

\- OR -

$\vec{r}_1 = \vec{a}_1 + \lambda\vec{d}_1 = (8, -19, 10) + \lambda(3, -16, 7)$

$\vec{r}_2 = \vec{a}_2 + \mu\vec{d}_2 = (15, 29, 5) + \mu(3, 8, -5)$ $\vec{d} = \vec{d}_1 \times \vec{d}_2$

$$\vec{l} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = \hat{i}((-16)(-5)-(7)(8)) - \hat{j}((3)(-5)-(7)(3)) + \hat{k}((3)(8)-(-16)(3))$$

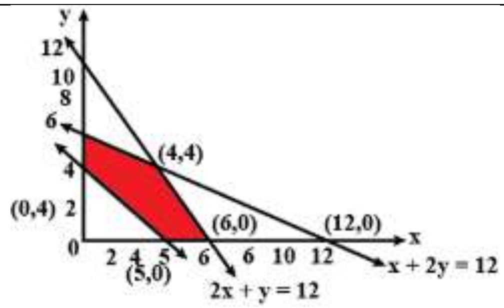
$$= \hat{i}(80-56) - \hat{j}(-15-21) + \hat{k}(24+48) = \hat{i}(24) - \hat{j}(-36) + \hat{k}(72) = (24, 36, 72)$$

$\vec{d} = (2, 3, 6)$

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + t(2\hat{i} + 3\hat{j} + 6\hat{k})$

(1+1+1)

30



Corner points	Z = 600x + 400y
(0,4)	1600 minimum
(0.6)	2400
(4,4)	4000 maximum
(6,0)	3600
(5,0)	3000

(1.5+1+0.5)

31

$P(G|\overline{H}) = \dfrac{P(G \cap \overline{H})}{P(\overline{H})} = \dfrac{1}{3}$

P(exactly one of them selected) $= P(R)\times P(\overline{J})\times P(\overline{A}) + P(\overline{R})\times P(J)\times P(\overline{A}) + P(\overline{R})\times P(\overline{J})\times P(A)$

$= \dfrac{6+12+8}{60} = \dfrac{13}{30}$

(1.5+1.5)

32	$d = \left\|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{\|\vec{b}_1 \times \vec{b}_2\|}\right\|$ Comparing the given equations with the equations $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ respectively, we have $\vec{a}_1 = 4\hat{i} - \hat{j}, \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k},\ \vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$ Now, $\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 0\hat{j} + 2\hat{k}$ and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j} + 0\hat{k}$ $\therefore\ (\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (-3\hat{i} + 0\hat{j} + 2\hat{k})\cdot(2\hat{i} - \hat{j} + 0\hat{k}) = -6 + 0 + 0 = -6$ and $\|\vec{b}_1 \times \vec{b}_2\| = \sqrt{4+1+0} = \sqrt{5}$ $\therefore$ Shortest distance $= \left\|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{\|\vec{b}_1 \times \vec{b}_2\|}\right\| = \frac{6}{\sqrt{5}}$ units. (1+2+2)
33	(a) – (i) Let $I = \int_0^{\pi} \frac{x}{(1+\sin x)} dx$ … (i) Then, $I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)} dx = \int_0^{\pi} \frac{(\pi - x)}{(1+\sin x)} dx$ …(ii) $2I = \pi\int_0^{\pi} \frac{dx}{(1+\sin x)} = \pi\cdot\int_0^{\pi} \frac{1}{(1+\sin x)} \times \frac{(1-\sin x)}{(1-\sin x)} dx$ or $2I = \pi\int_0^{\pi}\left(\frac{1-\sin x}{\cos^2 x}\right) dx = \pi\cdot\left[\int_0^{\pi} \sec^2 x\, dx - \int_0^{\pi} \sec x \tan x\, dx\right] = \pi\cdot\{[\tan x]_0^{\pi} - [\sec x]_0^{\pi}\} = 2\pi$ $\therefore\ I = \pi$, i.e., $\int_0^{\pi} \frac{x}{(1+\sin x)} dx = \pi.$ (a) – (ii) $\int e^x \frac{(1-\sin x)}{(1-\cos x)} dx = \int e^x \frac{\left(1 - 2\sin\frac{x}{2}\cos\frac{x}{2}\right)}{2\sin^2\frac{x}{2}} dx = \int e^x \left(\frac{1}{2\sin^2\frac{x}{2}} - \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right) dx$ $= \int e^x \left[\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} + \left(-\cot\frac{x}{2}\right)\right] dx = \int e^x\,[f'(x) + f(x)]\,dx = e^x f(x) + c = \mathbf{-e^x \cdot \cot\frac{x}{2} + c}$ - OR - (1+2+2)

	$\int \frac{x^4}{(x-1)(x^2+1)}dx = \int \frac{x^4}{x^3-x^2+x-1}dx$ $\int \frac{x^4}{(x-1)(x^2+1)}dx = \int \left[x+1+\frac{1}{(x-1)(x^2+1)}\right]dx = \frac{x^2}{2}+x+\int \frac{1}{(x-1)(x^2+1)}dx$ $\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1}+\frac{Bx+C}{x^2+1}$ $\Rightarrow 1 = A(x^2+1)+(Bx+C)(x-1)$ $\Rightarrow 1 = (A+B)x^2+(A-C)+x(C-B)$ $\therefore\ A+B = 0 \Rightarrow A = -B$ $C-B = 0 \Rightarrow C = B \Rightarrow A-C = 1$ $\Rightarrow -2C = 1 \Rightarrow C = -\frac{1}{2} \Rightarrow B = -\frac{1}{2}$ and $A = \frac{1}{2}$ $\int \frac{1}{(x-1)(x^2+1)}dx = \frac{1}{2}\int\left[\frac{+1}{x-1}+\frac{-x-1}{x^2+1}\right]dx = \frac{1}{2}\left[\int\frac{dx}{x-1}-\int\frac{x}{x^2+1}dx-\int\frac{dx}{x^2+1}\right]$ $= \frac{1}{2}\left[\log\lvert x-1\rvert-\frac{1}{2}\log\lvert x^2+1\rvert-\tan^{-1}x\right]+c_1 = \frac{1}{2}\log\left\lvert\frac{x-1}{\sqrt{x^2+1}}\right\rvert-\frac{1}{2}\tan^{-1}x+c$ $= \frac{x^2}{2}+x+\frac{1}{2}\log\left\lvert\frac{x-1}{\sqrt{x^2+1}}\right\rvert-\frac{1}{2}\tan^{-1}x+c$ (1+1+2+1)
34	We have, $(x^2+y^2)\,dx - 2xy\,dy = 0 \Rightarrow \frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ ... (i) Let $f(x, y) = \frac{x^2+y^2}{2xy}$, so $f(\lambda x, \lambda y) = \frac{\lambda^2x^2+\lambda^2y^2}{2\lambda x\cdot\lambda y} = \frac{\lambda^2}{\lambda^2}f(x,y) = \lambda^0 f(x, y)$ $\therefore$ This is homogeneous differential equation. Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ So, equation (i) becomes $v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2x\cdot vx} = \frac{x^2(1+v^2)}{x^2\cdot 2v} = \frac{1+v^2}{2v}$ $\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v = \frac{1+v^2-2v^2}{2v} = \frac{1-v^2}{2v}$ Separating the variables, we get $\frac{2v}{1-v^2}\,dv = \frac{dx}{x}$ $\Rightarrow \int\frac{2v}{1-v^2}\,dv = \int\frac{dx}{x} \Rightarrow -\log\lvert 1-v^2\rvert = \log x + C \Rightarrow \log x + \log\lvert 1-v^2\rvert = -C$ $\Rightarrow \log x(1-v^2) = -C \Rightarrow x\left(1-\frac{y^2}{x^2}\right) = e^{-C} \Rightarrow x^2-y^2 = Ax$, where $e^{-C} = A$ Hence, $x^2 - y^2 = Ax$ is the general solution of the given differential equation. (1+2+2) - OR - $x\,dy - (y+2x^2)\,dx = 0 \Rightarrow x\,dy = (y+2x^2)\,dx$ $\Rightarrow \frac{dy}{dx} = \frac{y+2x^2}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x}+\frac{2x^2}{x} \Rightarrow \frac{dy}{dx}-\frac{y}{x} = 2x$ On comparing with $\frac{dy}{dx} + Py = Q$; Here 'P' $= \frac{-1}{x}$, $Q = 2x$ I.F. $= e^{\int P\,dx} = e^{\int\frac{-1}{x}dx} = e^{-\log\lvert x\rvert} = e^{\log\lvert x^{-1}\rvert} = x^{-1} = \frac{1}{x}$; (1+1+2+1)

	Hence the required solution is $y(\text{I.F.}) = \int Q(\text{I.F.})dx$ $\quad y\left(\frac{1}{x}\right) = \int 2x.\frac{1}{x}dx = \int 2\,dx \Rightarrow \frac{y}{x} = 2x + C$ $\Rightarrow \log\|y\| + \frac{1}{y} = -\frac{1}{x} + x + C$
35	$x + y + z = 12 \qquad 2x + 3y + 3z = 33 \qquad x - 2y + z = 0$ $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} \qquad \|A\| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{vmatrix}$ $= 1\,(3 + 6) - 1\,(2 - 3) + 1\,(-4 - 3) = 9 + 1 - 7 = 3$ $adj(A) = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$ $.\ A^{-1} = \frac{1}{\|A\|}(adjA) = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$ $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} \qquad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 108 - 99 \\ 12 + 0 + 0 \\ -84 + 99 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ $\qquad x = 3, y = 4, z = 5$ (1+1+2+1)
36	(*i*) Number of relations from B to $G = 2^6$ (*ii*) Number of functions from B to $G = 2^{n(B \times G)}$ $= (n(G))^{n(B)} = 2^3 = 8$ **Symmetric:** Let $(x, y) \in R$ $\Rightarrow$ x and y are of same sex. $\Rightarrow$ y and x are of same sex. $\Rightarrow$ $(y, x) \in R$ **Transitive:** Let (x, y) and $(y, z) \in R$ $\Rightarrow$ x and y are of same sex. and y and z are of same sex. $\Rightarrow$ x and z are of same sex. $\Rightarrow$ $(x, z) \in R \Rightarrow R$ is transitive. Hence R is reflexive, symmetric and transitive $\therefore$ R is an equivalence relation in B. (1+1+2) **OR** (*b*) $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$ $\because$ $f(b_1) = g_1$ and $f(b_3) = g_1$ $\Rightarrow$ $f(b_1) = f(b_3)$ but $b_1 \neq b_3$ As b_1 and b_3 represents two different boys. $\Rightarrow$ f is not one-one. $\Rightarrow$ f is not a bijective map.
37	(*i*) **Given.** Length = x, Breadth = y and Radius (r) = $\frac{y}{2}$. Perimeter of the floor = 200 m $\Rightarrow x + x + 2 \times \pi r = 200 \qquad \Rightarrow 2x + 2\pi\left(\frac{y}{2}\right) = 200 \qquad \therefore$ $\mathbf{2x + \pi y = 200}$ (*ii*) We have from point (*i*), $\pi y = 200 - 2x \qquad \Rightarrow y = \frac{200 - 2x}{\pi}$ ...(A) **As we know, area of Rectangular region** = $x.y$ $= x\left(\frac{200 - 2x}{\pi}\right) = \frac{2}{\pi}(100x - x^2)$ (1+1+2)

(iii) $A = \frac{2}{\pi}(100x - x^2)$ ...**[From point (*ii*)**

Differentiate both sides w.r.t x, we have $\frac{dA}{dx} = \frac{2}{\pi}(100 - 2x)$

When $\frac{dA}{dx} = 0, \Rightarrow \frac{2}{\pi}(100 - 2x) = 0 \Rightarrow 100 - 2x = 0 \quad \therefore \quad x = \frac{100}{2} = 50$

Again differentiating, $\frac{d^2A}{dx^2} = \frac{2}{\pi}(-2) = \frac{-4}{\pi}$

$\frac{d^2A}{dx^2}$ at $x = 50 = \frac{-4}{\pi} < 0$ (-ve); Hence A is maximum at $x = 50$.

$\therefore$ **Maximum value of Area A** $= \frac{2}{\pi}[100(50) - (50)^2] = \frac{2}{\pi}(5000 - 2500) = \mathbf{\frac{5000}{\pi}\ m^2}$

OR

Area of floor = Area of rectangle + Area of circle

$= \frac{2}{\pi}(100x - x^2) + \pi r^2$ ...**[As Area of Rectangle = $\frac{2}{\pi}(100x - x^2)$**

$= \frac{2}{\pi}(100x - x^2) + \pi\left(\frac{y^2}{4}\right)$ ...[$\because r = \frac{y}{2}$

Let $F = \frac{2}{\pi}(100x - x^2) + \frac{\pi}{4}\left(\frac{200 - 2x}{\pi}\right)^2$ ...**[From (A)**

Differentiating the above w.r.t. x, we get

$\frac{dF}{dx} = \frac{2}{\pi}(100 - 2x) + \frac{1}{4\pi} \times 2(200 - 2x) \times (-2) = \frac{200 - 4x - 200 + 2x}{\pi} = \frac{-2x}{\pi}$

When $\frac{dF}{dx} = 0, \frac{-2x}{\pi} = 0 \quad \therefore$ $\mathbf{x = 0\ m}$

38 (i) Let the probability that the product was made by Machine A be E_1. Let the probability that the product was made by Machine B be E_2. Let the probability that the product was defective be A.

$P(E_1) = 60/100 = 0.6$, $P(E_2) = 40/100 = 0.4$

2% of items produced by A was defective $P(A|E_1) = 2/100 = 0.02$

1% of items produced by B was defective $P(A|E_2) = 1/100 = 0.01$

Total Probability, $P(A) = P(E_1) \times P(A|E_1) + P(E_2) \times P(A|E_2) = 0.6 \times 0.02 + 0.4 \times 0.01 = 0.012 + 0.04 = 0.016$

(ii) Using Bayes' Theorem, $P(E_2|A) = \frac{P(E_2) \times P(A|E_2)}{P(E_1) \times P(A|E_1) + P(E_2) \times P(A|E_2)} = \frac{0.4 \times 0.01}{0.6 \times 0.02 + 0.4 \times 0.01} = 0.25$ (2+2)

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